2018

AP CollegeBoard

AP Calculus BC

Sample Student Responses and Scoring Commentary

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Question 1

(a) $\int_0^{300} r(t)\, dt = 270$ According to the model, 270 people enter the line for the escalator during the time interval $0 \le t \le 300$.	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(b) $20 + \int_0^{300} (r(t) - 0.7)\, dt = 20 + \int_0^{300} r(t)\, dt - 0.7 \cdot 300 = 80$ According to the model, 80 people are in line at time $t = 300$.	$2 : \begin{cases} 1 : \text{considers rate out} \\ 1 : \text{answer} \end{cases}$
(c) Based on part (b), the number of people in line at time $t = 300$ is 80. The first time t that there are no people in line is $300 + \frac{80}{0.7} = 414.286$ (or 414.285) seconds.	1 : answer
(d) The total number of people in line at time t, $0 \le t \le 300$, is modeled by $20 + \int_0^t r(x)\, dx - 0.7t$. $r(t) - 0.7 = 0 \Rightarrow t_1 = 33.013298,\ t_2 = 166.574719$	$4 : \begin{cases} 1 : \text{considers } r(t) - 0.7 = 0 \\ 1 : \text{identifies } t = 33.013 \\ 1 : \text{answers} \\ 1 : \text{justification} \end{cases}$

t	People in line for escalator
0	20
t_1	3.803
t_2	158.070
300	80

The number of people in line is a minimum at time $t = 33.013$ seconds, when there are 4 people in line.

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1. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

$$\int_0^{300} 44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 dt = \boxed{270}$$

(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

$.7(300) = 210$

$20 + 270 = 290$

$290 - 210 = \boxed{80}$

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(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

$(t-300)(.7) - 80 = 0$

$.7t - 210 - 80 = 0$

$.7t = +290$

$\boxed{t = 414.286s}$

(d) JFor $0 \leq t \leq 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

p= total people

$\frac{dp}{dt} = r(t) - .7$

$0 = r(t) - .7$

$t = 166.575$

$t = 33.013$

$p(t) = \int_0^t r(x) - .7 \, dx + 20$

t	$p(t)$
0	20
33.013	3.803
166.575	158.07014
300	80

minimum at time $t = 33.013s$ when 4 people are in line

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1. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

$S(0) = 20$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

$\int_0^{300} 44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 dt =$ 270 people enter the line for the escalator during the time interval $0 \le t \le 300$

(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

$L(t) = 0.7$

$20 + \int_0^{300} r(t)\,dt - \int_0^{300} 0.7\,dt$

$20 + 270 - 210$

At time $t = 300$ seconds, there are 80 people in line for the escalator.

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(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

For $t > 300$, the first time t that there are no people in line for the escalator is $t = 325$

(d) For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

$$44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 - 0.7$$

$$20 + \int_0^{33.013298 \to A} r(t)\,dt - \int_0^{A} 0.7\,dt$$

$$44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 = 0.7$$

At time $t = 33.013$ seconds there is a minimum of 3.803 people on the escalator.

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1C
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1. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

$r(t) = 44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 \quad (0,300)$

$\text{people} = \int_0^{300} 44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 dt$

people = $\boxed{\text{5670O people}}$

(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

$20 + \int_0^{300} r(t)\, dt - \left(\int_0^{300} (0.7\, dt)\right)$

$\left[20 + \int_0^{300} 44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 dt\right] - \int_0^{300} .7\, dt$

$= \boxed{\text{80 people}}$

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(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

$r(t) = 0 \qquad t > 300$

There are no people in line for the escalator first at time $t = 300$.

(d) For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

rate enter − rate exit = 0

* $r(t) = 0$ and changes inc → dec

$r(t) = 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 = 0$

$t = 150$ sec.

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Question 1

Overview

The context of this problem is a line of people waiting to get on an escalator. The function r models the rate at which people enter the line, where $r(t) = 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7$ for $0 \le t \le 300$, and $r(t) = 0$ for $t > 300$; $r(t)$ is measured in people per second, and t is measured in seconds. Further, it is given that people exit the line to get on the escalator at a constant rate of 0.7 person per second and that there are 20 people in the line at time $t = 0$. In part (a) students were asked how many people enter the line for the escalator during the time interval $0 \le t \le 300$. A correct response demonstrates the understanding that the number of people entering the line during this time interval is obtained by integrating the rate at which people enter the line across the time interval. Thus, this number is the value of the definite integral $\int_0^{300} r(t)\,dt$. A numerical value for this integral should be obtained using a graphing calculator. In part (b) students were given that there are always people in line during the time interval $0 \le t \le 300$ and were asked to determine the number of people in line at time $t = 300$. A correct response should take into account the 20 people in line initially, the number that entered the line as determined in part (a), and the number of people that exit the line to get on the escalator. It was given in the problem statement that people exit the line at a constant rate of 0.7 person per second, so the number of people that exit the line to get on the escalator can be found by multiplying this constant rate times the duration of the interval, namely 300 seconds. In part (c) students were asked for the first time t beyond $t = 300$ when there are no people in line for the escalator. Because no more people join the line after $t = 300$ seconds, and people exit the line at the constant rate of 0.7 person per second, dividing the answer to part (b) by 0.7 gives the number of seconds beyond $t = 300$ before the line empties for the first time. Adding this quotient to 300 produces the answer. In part (d) students were asked when, during the time interval $0 \le t \le 300$, is the number of people in line a minimum, and to determine the number of people in line (to the nearest whole number) at that time, with the added admonition to justify their answer. The Extreme Value Theorem guarantees that the number of people in line at time t, given by the expression $20 + \int_0^t r(x)\,dx - 0.7t$, attains a minimum on the interval $0 \le t \le 300$. Correct responses should demonstrate that the rate of change of the number of people in line is given by $r(t) - 0.7$. Solving for $r(t) - 0.7 = 0$ within the interval $0 < t < 300$ yields two critical points, t_1 and t_2, so candidates for the time when the line is a minimum are $t = 0$, t_1, t_2, and $t = 300$. The number of people in line at times t_1 and t_2 is computed from $20 + \int_0^{t_1} r(x)\,dx - 0.7t_1$ and $20 + \int_0^{t_2} r(x)\,dx - 0.7t_2$. The answer is the least of 20, these two computed values (to the nearest whole number), and the answer to part (b), together with the corresponding time t for this minimum value.

For part (a) see LO 3.3B(b)/EK 3.3B2, LO 3.4A/EK 3.4A2, LO 3.4E/EK 3.4E1. For parts (b) and (c), see LO 3.4A/EK 3.4A2, LO 3.4E/EK 3.4E1. For part (d) see LO 1.2B/EK 1.2B1, LO 2.3C/EK 2.3C3, LO 3.3B(b)/EK 3.3B2, LO 3.4A/EK 3.4A2, LO 3.4E/EK 3.4E1. This problem incorporates the following Mathematical Practices for AP Calculus (MPACs): reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, building notational fluency, and communicating.

Sample: 1A
Score: 9

The response earned all 9 points: 2 points in part (a), 2 points in part (b), 1 point in part (c), and 4 points in part (d). In part (a) the response earned the first point for $\int_0^{300} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 dt$. The response earned the

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Question 1 (continued)

second point for the answer 270. In part (b) the response earned the first point for -210 on the right side of the response area. The response earned the second point for the answer 80. In part (c) the response earned the point for the answer 414.286. What appears to be a fourth digit of 5 is not a digit but the letter s for seconds. Units are not required to earn any points in this question. In part (d) the response earned the first point for $0 = r(t) - .7$ in line 2. The response earned the second point with $t = 33.013$ in line 4. The response earned the third point for the boxed information. What appears to be a fourth digit of 5 is not a digit but the letter s for seconds. The response earned the fourth point for the candidates test demonstrated with the table. The expression for the function p, identified as "total people," supports how the values at 33.013 and 166.575 are produced.

Sample: 1B
Score: 6

The response earned 6 points: 2 points in part (a), 2 points in part (b), no point in part (c), and 2 points in part (d). In part (a) the response earned the first point for $\int_0^{300} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7\,dt.$ The response earned the second point for the answer 270. In part (b) the response earned the first point with the term $-\int_0^{300} 0.7\,dt.$ The response earned the second point for the answer 80. In part (c) the response did not earn the point because what is presented is incorrect. In part (d) the response earned the first point in line 2 with the equation $44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 = 0.7.$ The response earned the second point with $t = 33.013$ in line 3. The response did not earn the third point for the answers because 3.803 is not rounded to a whole number. The response did not earn the fourth point because it does not have a complete justification.

Sample: 1C
Score: 3

The response earned 3 points: 1 point in part (a), 2 points in part (b), no point in part (c), and no points in part (d). In part (a) the response earned the first point for $\int_0^{300} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7\,dt.$ The response did not earn the second point because 56700 is incorrect. In part (b) the response earned the first point with the term $-\left(\int_0^{300}(0.7\,dt)\right)$ in line 1. The response earned the second point for the answer 80. In part (c) the response did not earn the point because what is presented is incorrect. In part (d) the response did not earn the first point with any of the equations presented. The equation at the top right "rate enter $-$ rate exit $= 0$" is too formulaic and not specific to the question. The response does not identify $t = 33.013$ and did not earn the second point. As a result, the response is not eligible to earn the remaining 2 points.

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Sample Student Responses and Scoring Commentary

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Question 2

(a) $p'(25) = -1.179$

At a depth of 25 meters, the density of plankton cells is changing at a rate of -1.179 million cells per cubic meter per meter.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{meaning with units} \end{cases}$

(b) $\int_0^{30} 3p(h)\,dh = 1675.414936$

There are 1675 million plankton cells in the column of water between $h = 0$ and $h = 30$ meters.

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $\int_{30}^{K} 3f(h)\,dh$ represents the number of plankton cells, in millions, in the column of water from a depth of 30 meters to a depth of K meters.

The number of plankton cells, in millions, in the entire column of water is given by $\int_0^{30} 3p(h)\,dh + \int_{30}^{K} 3f(h)\,dh.$

Because $0 \le f(h) \le u(h)$ for all $h \ge 30$,

$$3\int_{30}^{K} f(h)\,dh \le 3\int_{30}^{K} u(h)\,dh \le 3\int_{30}^{\infty} u(h)\,dh = 3 \cdot 105 = 315.$$

The total number of plankton cells in the column of water is bounded by $1675.415 + 315 = 1990.415 \le 2000$ million.

$3 : \begin{cases} 1 : \text{integral expression} \\ 1 : \text{compares improper integral} \\ 1 : \text{explanation} \end{cases}$

(d) $\int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt = 757.455862$

The total distance traveled by the boat over the time interval $0 \le t \le 1$ is 757.456 (or 757.455) meters.

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{total distance} \end{cases}$

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2A
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2. Researchers on a boat are investigating plankton cells in a sea. At a depth of h meters, the density of plankton cells, in millions of cells per cubic meter, is modeled by $p(h) = 0.2h^2 e^{-0.0025h^2}$ for $0 \le h \le 30$ and is modeled by $f(h)$ for $h \ge 30$. The continuous function f is not explicitly given.

(a) Find $p'(25)$. Using correct units, interpret the meaning of $p'(25)$ in the context of the problem.

$p'(25) = -1.179$

at a depth of 25 meters, the density of plankton is decreasing at a rate of $1.179 \frac{\text{millions of cells}}{\text{meter}^3} / \text{meter}$ as depth increases

(b) Consider a vertical column of water in this sea with horizontal cross sections of constant area 3 square meters. To the nearest million, how many plankton cells are in this column of water between $h = 0$ and $h = 30$ meters?

$3 \int_0^{30} p(h)\,dh = 1675$ million plankton cells

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me 1990

(c) There is a function u such that $0 \le f(h) \le u(h)$ for all $h \ge 30$ and $\int_{30}^{\infty} u(h)\,dh = 105$. The column of water in part (b) is K meters deep, where $K > 30$. Write an expression involving one or more integrals that gives the number of plankton cells, in millions, in the entire column. Explain why the number of plankton cells in the column is less than or equal to 2000 million.

number of plankton cells $= 3\int_0^{30} f(h)\,dh + 3\int_{30}^{K} f(h)\,dh$

since K is finite, and $0 \le f(h) \le u(h)$ for all $h \ge 30$,

$3\int_{30}^{K} f(h)\,dh \le 3\int_{30}^{\infty} u(h)\,dh = 315$

since $3\int_0^{30} f(h)\,dh$ is approximately 1675,

$3\int_0^{30} f(h)\,dh + 3\int_{30}^{K} f(h)\,dh \le 1990$ million cells

---

(d) The boat is moving on the surface of the sea. At time $t \ge 0$, the position of the boat is $(x(t), y(t))$, where $x'(t) = 662\sin(5t)$ and $y'(t) = 880\cos(6t)$. Time t is measured in hours, and $x(t)$ and $y(t)$ are measured in meters. Find the total distance traveled by the boat over the time interval $0 \le t \le 1$.

$\int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt = $! $\boxed{757.456 \text{ meters}}$

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2B
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2. Researchers on a boat are investigating plankton cells in a sea. At a depth of h meters, the density of plankton cells, in millions of cells per cubic meter, is modeled by $p(h) = 0.2h^2e^{-0.0025h^2}$ for $0 \le h \le 30$ and is modeled by $f(h)$ for $h \ge 30$. The continuous function f is not explicitly given.

(a) Find $p'(25)$. Using correct units, interpret the meaning of $p'(25)$ in the context of the problem.

$$p'(h) = (.4x - .001x^3)(.99750)^{x^2}$$

$$p'(25) = -1.18$$

At a depth of 25 meters, the density of plankton is decreasing by 1.18 million cells per cubic meters per meter in depth.

(b) Consider a vertical column of water in this sea with horizontal cross sections of constant area 3 square meters. To the nearest million, how many plankton cells are in this column of water between $h = 0$ and $h = 30$ meters?

$$3\int_0^{30} .2h^2 e^{-.0025h^2}\,dh$$

$\approx$ 1,675 million plankton cells

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(c) There is a function u such that $0 \le f(h) \le u(h)$ for all $h \ge 30$ and $\int_{30}^{\infty} u(h)\, dh = 105$. The column of water in part (b) is K meters deep, where $K > 30$. Write an expression involving one or more integrals that gives the number of plankton cells, in millions, in the entire column. Explain why the number of plankton cells in the column is less than or equal to 2000 million.

$$3\int_0^{30} p(h)\,dh + 3\int_{30}^{K} f(h)\,dh$$

$$1{,}675 + 3\int_{30}^{K} f(h)\,dh$$

Since $f(h)$ is less than $u(h)$, $\int_{30}^{\infty} f(h)\,dh$ is less than $\int_{30}^{\infty} u(h)\,dh$. Therefore $\int_{30}^{\infty} f(h)\,dh$ is less than 105.

$1{,}675 + 105 < 2000$

(d) The boat is moving on the surface of the sea. At time $t \ge 0$, the position of the boat is $(x(t), y(t))$, where $x'(t) = 662\sin(5t)$ and $y'(t) = 880\cos(6t)$. Time t is measured in hours, and $x(t)$ and $y(t)$ are measured in meters. Find the total distance traveled by the boat over the time interval $0 \le t \le 1$.

$$\int_0^1 \sqrt{(662\sin(5t))^2 + (880\cos(6t))^2}\,dt = 757.46 \text{ meters}$$

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2C
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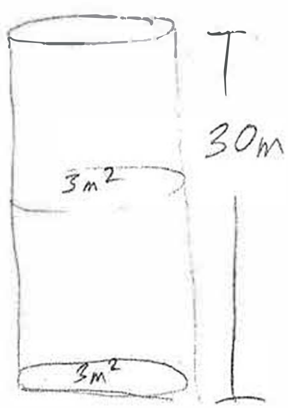
2. Researchers on a boat are investigating plankton cells in a sea. At a depth of h meters, the density of plankton cells, in millions of cells per cubic meter, is modeled by $p(h) = 0.2h^2e^{-0.0025h^2}$ for $0 \le h \le 30$ and is modeled by $f(h)$ for $h \ge 30$. The continuous function f is not explicitly given.

(a) Find $p'(25)$. Using correct units, interpret the meaning of $p'(25)$ in the context of the problem.

$p'(25) = \frac{d}{dh}\left(0.2h^2e^{-0.0025h^2}\right) = \boxed{-1.179 \text{ million cells}/m^3/s}$

$p'(25)$ is the rate of change of the density of the plankton. In this case, the density of the plankton is decreasing at a rate of 1.179 million cells/m³/s.

(b) Consider a vertical column of water in this sea with horizontal cross sections of constant area 3 square meters. To the nearest million, how many plankton cells are in this column of water between $h = 0$ and $h = 30$ meters?



#cells $= \int_0^{30} p(h)\,dh = \boxed{558 \text{ million cells}}$

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(c) There is a function u such that $0 \le f(h) \le u(h)$ for all $h \ge 30$ and $\int_{30}^{\infty} u(h)\, dh = 105$. The column of water in part (b) is K meters deep, where $K > 30$. Write an expression involving one or more integrals that gives the number of plankton cells, in millions, in the entire column. Explain why the number of plankton cells in the column is less than or equal to 2000 million.

$$\int_0^{30} p(h)\, dh + \int_{30}^{\infty} u(h)\, dh = \text{\# plankton cells in column}$$

(d) The boat is moving on the surface of the sea. At time $t \ge 0$, the position of the boat is $(x(t), y(t))$, where $x'(t) = 662\sin(5t)$ and $y'(t) = 880\cos(6t)$. Time t is measured in hours, and $x(t)$ and $y(t)$ are measured in meters. Find the total distance traveled by the boat over the time interval $0 \le t \le 1$.

$$\text{dist} = \int_0^1 \sqrt{(880\cos(6t))^2 + (662\sin(5t))^2}\, dt$$

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Overview

The context of this problem is an investigation of plankton cells in a sea. The density of plankton cells at a depth of h meters is modeled by $p(h) = 0.2h^2e^{-0.0025h^2}$ for $0 \le h \le 30$ and is modeled by $f(h)$ for $h \ge 30$. The density is measured in millions of cells per cubic meter, and the function f is stated to be continuous but is not explicitly given. In part (a) students were asked for the value of $p'(25)$ and to interpret the meaning of $p'(25)$ in the context of the problem. A correct response should give the derivative value as obtained from a graphing calculator and interpret this value as the rate of change of the density of plankton cells, in million cells per cubic meter per meter, at a depth of 25 meters. In part (b) students were asked for the number of plankton cells (to the nearest million) contained in the top 30 meters of a vertical column of water that has horizontal cross sections of constant area 3 square meters. A correct response should combine the density of the plankton, $p(h)$ million cells per cubic meter, and the cross-sectional area of the vertical column to obtain that the number of plankton cells changes at a rate of $3p(h)$ million cells per meter of depth. Thus the number of plankton cells (in millions) in the top 30 meters of the column is the accumulation of this rate for $0 \le h \le 30$, given by the integral $\int_0^{30} 3p(h)\,dh$. This integral should be evaluated using a graphing calculator and rounded to the nearest integer. In part (c) a function u is introduced that satisfies $0 \le f(h) \le u(h)$ for $h \ge 30$ and $\int_{30}^{\infty} u(h)\,dh = 105$. Given that the column of water in part (b) is K meters deep, where $K > 30$, students were asked to write an expression involving one or more integrals that gives the number of plankton cells, in millions, in the entire column, and to explain why the number of plankton cells in the column is at most 2000 million. Using the idea from part (b), a correct response should realize the number of plankton cells in the column is a definite integral of 3 times the density from $h = 0$ to $h = K$. Because $K > 30$, and the density is given by $f(h)$ at depths $h \ge 30$, the number of plankton cells, in millions, in the entire column is $\int_0^{30} 3p(h)\,dh + \int_{30}^{K} 3f(h)\,dh$. The first term was found in part (b); the second term can be bounded by $3 \cdot 105 = 315$ using the given information about the functions f and u, together with properties of integrals. Summing the answer from part (b) with the upper bound of 315 for the second term shows that the number of plankton cells in the entire column of water is less than 2000 million. In part (d) the position of a research boat on the sea's surface is described parametrically by $(x(t), y(t))$ for $t \ge 0$, where $x'(t) = 662\sin(5t)$, $y'(t) = 880\cos(6t)$, t is measured in hours, and $x(t)$ and $y(t)$ are measured in meters. Students were asked to find the total distance traveled by the boat over the time interval $0 \le t \le 1$.
A correct response should find the total distance traveled by the boat as the integral of its speed, $\sqrt{(x'(t))^2 + (y'(t))^2}$, across the time interval $0 \le t \le 1$ and evaluate this integral using a graphing calculator.

For part (a) see LO 2.3A/EK 2.3A1, LO 2.3D/EK 2.3D1. For part (b) see LO 3.3B(b)/EK 3.3B2, LO 3.4E/EK 3.4E1. For part (c) see LO 3.2C/EK 3.2C2, LO 3.2D (BC)/EK 3.2D2 (BC), LO 3.4E/EK 3.4E1. For part (d) see LO 3.3B(b)/EK 3.3B2, LO 3.4C/EK 3.4C2 (BC). This problem incorporates the following Mathematical Practices for AP Calculus (MPACs): reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, building notational fluency, and communicating.

Sample: 2A
Score: 9

The response earned all 9 points: 2 points in part (a), 2 points in part (b), 3 points in part (c), and 2 points in part (d). In part (a) the response earned the first point for presenting -1.179. The response earned the second point by addressing the value of 25 and identifying that the density is decreasing at a rate of 1.179 millions of

Question 2 (continued)

cells per cubic meter per meter. In part (b) the response earned the first point for presenting a definite integral with the correct integrand. The response earned the second point for computing 1675. In part (c) the response earned the first point for presenting the correct integral expression for the number of plankton cells, in millions, in line 1. The response earned the second point for comparing $3\int_{30}^{K} f(h)\,dh$ to $3\int_{30}^{\infty} u(h)\,dh.$ The response earned the third point for the explanation with an upper bound of 1990. In part (d) the response earned the first point for the integral setup for the distance traveled. The response earned the second point for the answer presented accurately to three places after the decimal point.

Sample: 2B
Score: 6

The response earned 6 points: no points in part (a), 2 points in part (b), 2 points in part (c), and 2 points in part (d). In part (a) the response presents $-1.18,$ the answer to only two decimal places, and did not earn the first point. Note that the symbolic expression for $p'(h)$ is correct; the point would still be earned with the use of x instead of h if the numerical answer were correct. The meaning presented does not express the concept of rate explicitly, so the second point was not earned. In part (b) the response earned the first point for presenting a definite integral with the correct integrand. The response earned the second point for computing 1,675. In part (c) the response earned the first point for presenting the correct integral expression for the number of plankton cells, in millions, in line 1. The response earned the second point for "$\int_{30}^{\infty} f(h)\,dh$ is less than $\int_{30}^{\infty} u(h)\,dh$" in line 4. The bound of 105 does not address the factor of 3 that is needed for the second integral in line 4, so the third point was not earned. In part (d) the response earned the first point for the integral setup for the distance traveled. Because the first point in part (a) was not earned due to a decimal presentation error, the second point in part (d) was earned, even though the answer of 757.46 is presented accurately to only 2 places after the decimal point. In any response, no more than 1 point may be impacted by decimal presentation errors.

Sample: 2C
Score: 3

The response earned 3 points: 1 point in part (a), 1 point in part (b), no points in part (c), and 1 point in part (d). In part (a) the response earned the first point for presenting $-1.179.$ The meaning presents incorrect units, so the second point was not earned. In part (b) the response earned the first point for presenting a definite integral with the correct integrand. The response does not address the factor of 3 that is needed, so the second point was not earned. In part (c) the response presents an incorrect integral expression, so the first point was not earned. There is no additional work, so neither of the last 2 points in part (c) were earned. In part (d) the response earned the first point for the integral setup for the distance traveled. The absolute values in the integrand are acceptable. Because the integral is not evaluated, the second point was not earned.

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AP Calculus BC

Sample Student Responses and Scoring Commentary

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Question 3

(a) $f(-5) = f(1) + \int_1^{-5} g(x)\,dx = f(1) - \int_{-5}^{1} g(x)\,dx$ $= 3 - \left(-9 - \frac{3}{2} + 1\right) = 3 - \left(-\frac{19}{2}\right) = \frac{25}{2}$	$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$
(b) $\int_1^6 g(x)\,dx = \int_1^3 g(x)\,dx + \int_3^6 g(x)\,dx$ $= \int_1^3 2\,dx + \int_3^6 2(x-4)^2\,dx$ $= 4 + \left[\frac{2}{3}(x-4)^3\right]_{x=3}^{x=6} = 4 + \frac{16}{3} - \left(-\frac{2}{3}\right) = 10$	$3: \begin{cases} 1: \text{split at } x = 3 \\ 1: \text{antiderivative of } 2(x-4)^2 \\ 1: \text{answer} \end{cases}$
(c) The graph of f is increasing and concave up on $0 < x < 1$ and $4 < x < 6$ because $f'(x) = g(x) > 0$ and $f'(x) = g(x)$ is increasing on those intervals.	$2: \begin{cases} 1: \text{intervals} \\ 1: \text{reason} \end{cases}$
(d) The graph of f has a point of inflection at $x = 4$ because $f'(x) = g(x)$ changes from decreasing to increasing at $x = 4$.	$2: \begin{cases} 1: \text{answer} \\ 1: \text{reason} \end{cases}$

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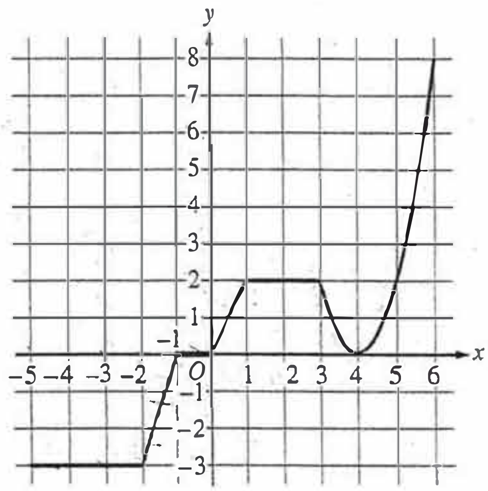

Graph of g

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$?

$f'(x) = g(x)$

$f(x) = \int_1^x g(t)\,dt + 3$

$f(-5) = -\int_{-5}^1 g(t)\,dt + 3$

$f(-5) = -\left(\frac{1}{2}(1)(2) - \frac{1}{2}(1)(3) - (3)(3)\right) + 3$

$f(-5) = -(1 - \frac{3}{2} - 9) + 3$

$f(-5) = 8 + \frac{3}{2} + 3$

$f(-5) = 11 + \frac{3}{2}$

(b) Evaluate $\int_1^6 g(x)\,dx$.

$18/3 = 6$

$\int_1^6 g(x)\,dx = \int_1^3 g(x)\,dx + \int_3^6 g(x)\,dx$

$\int_1^6 g(x)\,dx = (2)(2) + \int_3^6 2(x-4)^2\,dx$

$\int_1^6 g(x)\,dx = 4 + \left(\frac{2}{3}(x-4)^3\right)\Big|_3^6$

$\int_1^6 g(x)\,dx = 4 + \left(\frac{2}{3}(8) - \frac{2}{3}(-1)\right)$

$\int_1^6 g(x)\,dx = 4 + \frac{16}{3} + \frac{2}{3}$

$\int_1^6 g(x)\,dx = 10$

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(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

On the interval $(0,1) \cup (4,6)$ f is both increasing and concave up since $f'(x) = g(x)$ and g is positive on the that interval meaning f is increasing on that interval, and g is increasing ~~on~~ that interval, meaning $f''(x) > 0$ on that interval, therefore f is concave up on that interval

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer. A

f has a point of inflection at $x=4$ since $f'(x) = g(x)$ and since g switches from decreasing to increasing at $x=4$, therefore $f''(4) = 0$ at that point and would change signs from $\ominus$ to $\oplus$ at $x=4$, therefore $x=4$ is an inflection point.

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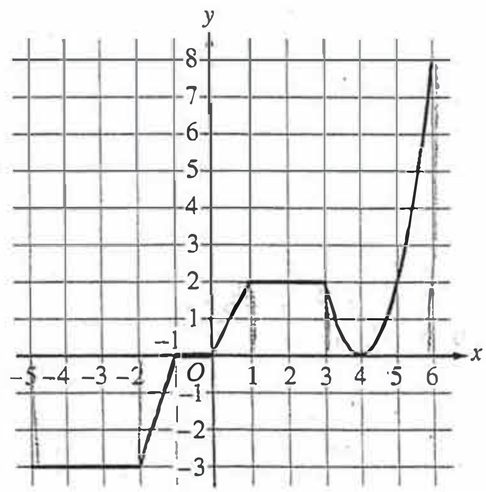

Graph of g

$g(x) = f'(x)$ $\quad f(x) = \int g(x)dx$

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$?

$\frac{6}{2} + \frac{19}{2} \quad -\frac{19}{2} - \frac{3}{2}$

$$\int_{-5}^{1} g(x)\,dx = f(1) - f(-5)$$
$$f(-5) = 3 - \int_{-5}^{1} g(x)\,dx$$
$$= 3 - \left[\left(-3(3) - \tfrac{1}{2}(1)(3)\right) + \tfrac{1}{2}(1)(2)\right]$$
$$= 3 - \left[\left(-9 - \tfrac{3}{2}\right) + 1\right]$$
$$= 3 - \left[\tfrac{-21}{2} + \tfrac{2}{2}\right] = 3 + \tfrac{19}{2} = \tfrac{25}{2}$$

(b) Evaluate $\int_1^6 g(x)\,dx$.

$u = x - 4$
$du = dx$

$$\int_1^6 g(x)dx = 3(3) + \int_3^6 g(x)\,dx$$
$$= 9 + \int_3^6 2(x-4)^2\,dx \qquad \frac{27}{3}$$
$$= 9 + 2\int_{-1}^{2} u^2\,du$$
$$= 9 + 2\cdot \tfrac{1}{3}u \Big|_{-1}^{2}$$
$$= 9 + 2\left(\tfrac{2}{3} - \tfrac{1}{3}\right) = 9 + \tfrac{2}{3} = \tfrac{29}{3}$$

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(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

f' is pos f'' pos

f is Increasing when $f'(x) = g(x)$ is positive
f is Concave up when $f'(x) = g(x)$ is increasing

$\Rightarrow$ The graph of f is concave up and increasing on $(0,1) \cup (4,6)$.

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer. a

f has a point of inflection when $f'(x) = g(x)$ has a maximum or minimum (local)

$\Rightarrow x = 4$

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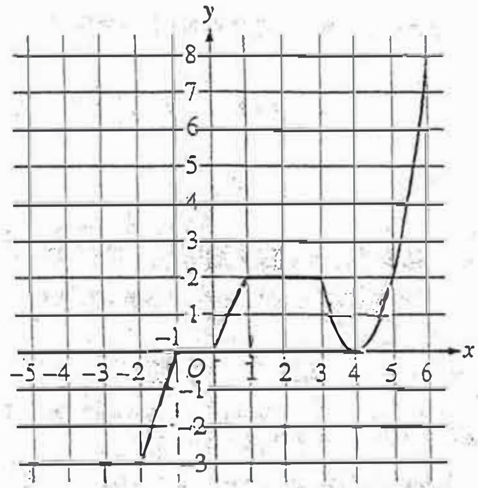

Graph of g $f'(x)$ Œ

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$? $f'(x) = g(x)$

$f'(x) = 2(x-4)^2$

$f(x) = \frac{2(x-4)^3}{3}$

$f(-5) = \frac{2(-5-4)^3}{3}$

(b) Evaluate $\int_1^6 g(x)\,dx$.

$\int_1^6 g(x)\,dx$

$\left.\frac{2(x-4)^3}{3}\right]_1^6$ $\frac{16}{3} + \frac{54}{3} = \boxed{\frac{70}{3}}$

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(c) bFor $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

from $0 < x < 1$ and $4 < x < 6$ the graph of f is both inc. and concave up because f' is above the x-axis (positive) and has an increasing slope

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

there is a point of inflection at $x = 4$ because the slope of f' changes from decreasing to increasing
(-) (+)

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2018 SCORING COMMENTARY

Question 3

Overview

In this problem the graph of the continuous function g is provided; g is piecewise linear for $-5 \le x < 3,$ and $g(x) = 2(x-4)^2$ for $3 \le x \le 6.$ It is also given that g is the derivative of the function f. In part (a) students were given that $f(1) = 3$ and asked for the value of $f(-5).$ A correct response should demonstrate knowledge that f is an antiderivative of g, so that $f(-5) = f(1) + \int_1^{-5} g(x)\,dx.$ The integral $\int_1^{-5} g(x)\,dx$ should then be evaluated using properties of definite integrals and computation of areas of the regions between the graph of g and the x-axis using geometry. In part (b) students were asked to evaluate $\int_1^6 g(x)\,dx.$ A correct response should use the property of integrals to split the interval of integration into the sum of integrals across adjacent intervals $[1, 3]$ and $[3, 6].$ One of the resulting integrals can be computed using geometry and the other using an antiderivative of $g(x) = 2(x-4)^2$ on the interval $3 \le x \le 6.$ In part (c) students were asked for the open intervals on $-5 < x < 6$ where the graph of f is both increasing and concave up and to give a reason for their answer. A correct response should demonstrate the connection between properties of the derivative of f and the properties of monotonicity and concavity for the graph of f. The graph of f is strictly increasing where $g = f'$ is positive, and the graph of g is concave up where the graph of $g = f'$ is increasing. In part (d) students were asked for the x-coordinate of each point of inflection of the graph of f and to give a reason for their answer. A correct response should convey that a point of inflection of the graph of f occurs at a point where the derivative of f changes from increasing to decreasing, or from decreasing to increasing. This can be obtained from the supplied graph of $g = f',$ which changes from decreasing to increasing at $x = 4.$

For part (a) see LO 3.2C/EK 3.2C1, LO 3.2C/EK 3.2C2. For part (b) see LO 3.2C/EK 3.2C1, LO 3.2C/EK 3.2C2, LO 3.3B(b)/EK 3.3B2, LO 3.3B(b)/EK 3.3B5. For parts (c) and (d), see LO 2.2A/EK 2.2A1. This problem incorporates the following Mathematical Practices for AP Calculus (MPACs): reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, connecting multiple representations, building notational fluency, and communicating.

Sample: 3A
Score: 9

The response earned all 9 points: 2 points in part (a), 3 points in part (b), 2 points in part (c), and 2 points in part (d). In part (a) the response earned the first point with the expression $-\int_{-5}^{1} g(t)\,dt$ in line 3 on the left. The second point would have been earned by the numerical expression in line 4 with no simplification. In this case, correct simplification to $11 + \frac{3}{2}$ earned the second point. In part (b) the response earned the first point with the sum of the two integrals in line 1 on the left. The second point was earned with the antiderivative expression $\left(\frac{2}{3}(x-4)^3\right)$ in line 3. The second point would have been earned by the numerical expression in line 4 with no simplification. In this case, correct simplification to 10 earned the third point. In part (c) the union of intervals "$(0, 1) \cup (4, 6)$" earned the first point. The second point was earned with the reason $f'(x) = g(x),$ "g is positive," and "g is increasing on that interval." In part (d) the first point was earned by identifying the x-coordinate of a point of inflection at $x = 4.$ The second point was earned with the reason $f'(x) = g(x)$ and "g switches from decreasing to increasing at $x = 4.$"

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Question 3 (continued)

Sample: 3B
Score: 6

The response earned 6 points: 2 points in part (a), no points in part (b), 2 points in part (c), and 2 points in part (d). In part (a) the response earned the first point with the expression $\int_{-5}^{1} g(x)\,dx$ in line 1. The second point would have been earned by the numerical expression in line 3 with no simplification. In this case, correct simplification to $\frac{25}{2}$ earned the second point. In part (b) the integral expression $3(3) + \int_{3}^{6} g(x)\,dx$ in line 1 did not earn the first point because $\int_{1}^{3} g(x)\,dx = 4,$ not 9. This response used substitution of variables to write $\int_{3}^{6} 2(x-4)^2\,dx$ in an equivalent form. The antiderivative in line 4 is incorrect, and the response did not earn the second point. As a result of this error, the response is not eligible for the answer point. In part (c) the union of intervals "$(0, 1) \cup (4, 6)$" earned the first point. The second point was earned with the reason "$f'(x) = g(x)$ is positive" in line 1 and "$f'(x) = g(x)$ is increasing" in line 2. In part (d) the first point was earned by identifying the x-coordinate of a point of inflection at $x = 4.$ The second point was earned with the reason "$f'(x) = g(x)$ has a maximum or minimum (local)."

Sample: 3C
Score: 3

The response earned 3 points: no points in part (a), 1 point in part (b), 1 point in part (c), and 1 point in part (d). In part (a) an integral expression is not presented nor is its numerical equivalent, so the first point was not earned. The value given for $f(-5)$ is incorrect, so the second point was not earned. In part (b) the response did not earn the first point because $\int_{1}^{6} g(x)\,dx$ is not written as the sum of two integrals or the equivalent. The second point was earned with the antiderivative expression in line 2. Because the second point was earned, the response is eligible for the third point. The answer is incorrect, however, so the third point was not earned. In part (c) the first point was earned with the intervals "$0 < x < 1$ and $4 < x < 6.$" Although "f' is above the x-axis" is a valid reason for why f is increasing on those intervals, f' "has an increasing slope" is not a valid reason to explain why the graph of f is concave up on those intervals. The second point was not earned. In part (d) the first point was earned by identifying the x-coordinate of a point of inflection at $x = 4.$ "The slope of f' changes from decreasing to increasing" is not a valid reason to explain why the graph of f has a point of inflection at $x = 4,$ so the second point was not earned.

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AP Calculus BC

Sample Student Responses and Scoring Commentary

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Question 4

(a) $H'(6) \approx \dfrac{H(7) - H(5)}{7 - 5} = \dfrac{11 - 6}{2} = \dfrac{5}{2}$

$H'(6)$ is the rate at which the height of the tree is changing, in meters per year, at time $t = 6$ years.

$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{interpretation with units} \end{cases}$

(b) $\dfrac{H(5) - H(3)}{5 - 3} = \dfrac{6 - 2}{2} = 2$

Because H is differentiable on $3 \le t \le 5$, H is continuous on $3 \le t \le 5$.

By the Mean Value Theorem, there exists a value c, $3 < c < 5$, such that $H'(c) = 2$.

$2 : \begin{cases} 1 : \dfrac{H(5) - H(3)}{5 - 3} \\ 1 : \text{conclusion using Mean Value Theorem} \end{cases}$

(c) The average height of the tree over the time interval $2 \le t \le 10$ is given by $\dfrac{1}{10 - 2}\displaystyle\int_2^{10} H(t)\, dt$.

$$\frac{1}{8}\int_2^{10} H(t)\, dt \approx \frac{1}{8}\left(\frac{1.5 + 2}{2} \cdot 1 + \frac{2 + 6}{2} \cdot 2 + \frac{6 + 11}{2} \cdot 2 + \frac{11 + 15}{2} \cdot 3\right)$$
$$= \frac{1}{8}(65.75) = \frac{263}{32}$$

The average height of the tree over the time interval $2 \le t \le 10$ is $\dfrac{263}{32}$ meters.

$2 : \begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{approximation} \end{cases}$

(d) $G(x) = 50 \Rightarrow x = 1$

$$\frac{d}{dt}(G(x)) = \frac{d}{dx}(G(x)) \cdot \frac{dx}{dt} = \frac{(1 + x)100 - 100x \cdot 1}{(1 + x)^2} \cdot \frac{dx}{dt} = \frac{100}{(1 + x)^2} \cdot \frac{dx}{dt}$$

$$\left.\frac{d}{dt}(G(x))\right|_{x=1} = \frac{100}{(1 + 1)^2} \cdot 0.03 = \frac{3}{4}$$

According to the model, the rate of change of the height of the tree with respect to time when the tree is 50 meters tall is $\dfrac{3}{4}$ meter per year.

$3 : \begin{cases} 2 : \dfrac{d}{dt}(G(x)) \\ 1 : \text{answer} \end{cases}$

Note: max $1/3$ [1-0] if no chain rule

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t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

4. The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

(a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

$$H'(6) \approx \frac{\Delta H(t)}{\Delta t} \approx \frac{H(7)-H(5)\,m}{(7-5)_{yr}} = \frac{(11-6)m}{(7-5)_y} = \boxed{\frac{5 \text{ meters}}{2 \text{ years}}}$$

When $t=6$ years, the rate at which the tree is growing is $H'(6)$ meters per year

(b) Explain why there must be at least one time t, for $2 < t < 10$, such that $H'(t) = 2$.

By the MVT, as $H(t)$ is continuous and differentiable on $t \in (2,10)$, there must be $H'(c)=2$ where $2<c<10$ if there exists $\frac{H(b)-H(a)}{b-a}=2$ on the interval $(2,10)$. $\frac{H(5)-H(3)}{(5-3)\text{years}} = \frac{6m-2m}{2\text{ years}} = 2\ m/yr$

So c exists on interval $c \in (2,10)$

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(c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \le t \le 10$.

Total height: $\frac{1}{2}\Big(1\,(1.5+2) + 2\,(2+6) + 2\,(6+11) + 3\,(11+15)\Big)$

Average height: $\frac{1}{10_y - 2_{yr}}$ total ht $= \boxed{\frac{1}{8} \times \frac{1}{2}\left(3.5 + 2(8) + 2(17) + 3(26)\right) \text{ meters}}$

(d) The height of the tree, in meters, can also be modeled by the function G, given by $G(x) = \frac{100x}{1+x}$, where x is the diameter of the base of the tree, in meters. When the tree is 50 meters tall, the diameter of the base of the tree is increasing at a rate of 0.03 meter per year. According to this model, what is the rate of change of the height of the tree with respect to time, in meters per year, at the time when the tree is 50 meters tall?

$G(x) = \frac{100x}{1+x}$ $\quad G(x) = 50 \quad \frac{dx}{dt} = 0.03$ m/y

$x = 1$ m

$G'(x) = \frac{100\frac{dx}{dt}(1+x) - \left(\frac{dx}{dt}\right)100x}{(1+x)^2}$

$\frac{100 \times 0.03(2) - 0.03 \times 100}{4}$

$50 = \frac{100x}{1+x} \Rightarrow \cancel{50}(1+x) = \cancel{100}^{2}x$

$\Rightarrow 1 = x \quad x = 1$ m

$\frac{6-3}{4} = \boxed{\frac{3}{4} \text{ m/year}}$

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t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

4. The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

(a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

$$H'(6) = \frac{H(7) - H(5)}{7-5} = \frac{11-6}{2} = \boxed{\frac{5}{2}\text{ meters/year}}$$

H'(6) is the rate that the tree is growing, in meters per year, at t = 6 years.

(b) Explain why there must be at least one time t, for $2 < t < 10$, such that $H'(t) = 2$.

H is twice-differentiable, which means it is also continuous. Therefore, the MVT guarantees that H'(t) = 2 since

$$\frac{H(10) - H(2)}{10-2} = 2$$

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(c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \leq t \leq 10$.

$$(1)\left(\frac{1.5+2}{2}\right)+(2)\left(\frac{2+6}{2}\right)+(2)\left(\frac{6+11}{2}\right)+(3)\left(\frac{11+15}{2}\right)$$

$$\frac{3.5}{2}+\frac{16}{2}+\frac{34}{2}+\frac{78}{2}$$

$$=\boxed{\frac{131.5}{2}\text{ meters}}$$

(d) The height of the tree, in meters, can also be modeled by the function G, given by $G(x) = \frac{100x}{1+x}$, where x is the diameter of the base of the tree, in meters. When the tree is 50 meters tall, the diameter of the base of the tree is increasing at a rate of 0.03 meter per year. According to this model, what is the rate of change of the height of the tree with respect to time, in meters per year, at the time when the tree is 50 meters tall?

$$\frac{dG}{dt}=\frac{(1+x)(100\frac{dx}{dt})-(100x)(\frac{dx}{dt})}{(1+x)^2}$$

$$\frac{(1+1)(100(0.03))-(100(1))(0.03)}{(1+1)^2}$$

$$\frac{(2)(3)-3}{4}=\frac{6-3}{4}=\boxed{\frac{3}{4}\text{ meters/year}}$$

$$50=\frac{100x}{1+x}$$
$$50(1+x)=100x$$
$$50+50x=100x$$
$$50=50x$$
$$x=1$$

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t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

4. The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

(a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

$$H'(6) = \frac{H(7) - H(5)}{7-5} = \frac{11-6}{7-5} \quad \frac{5}{2}$$

H'(6) is the rate in m/year in which the height of a tree increases.

(b) Explain why there must be at least one time t, for $2 < t < 10$, such that $H'(t) = 2$.

There must be one time t for 2 < t < 10 that H'(t) = 2 b/c H(t) is continuous and differentiable

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(c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \le t \le 10$.

$$\tfrac{1}{2}(1.5+2) + \tfrac{1}{2}(2+6)2 + \tfrac{1}{2}(6+11)2 + \tfrac{1}{2}(11+15)3$$

$$= \frac{3.5}{2} + 8 + 17 + 39$$

$$= \frac{7}{2} + \frac{16}{2} + \frac{34}{2} + \frac{78}{2}$$

$$= \frac{135}{2}$$

(d) The height of the tree, in meters, can also be modeled by the function G, given by $G(x) = \dfrac{100x}{1+x}$, where x is the diameter of the base of the tree, in meters. When the tree is 50 meters tall, the diameter of the base of the tree is increasing at a rate of 0.03 meter per year. According to this model, what is the rate of change of the height of the tree with respect to time, in meters per year, at the time when the tree is 50 meters tall?

$$r = \frac{d}{2}$$

$$G'(x) = \frac{(1+x)(100) - 100x}{(1+x)^2}$$

$$A = bh + \pi r^2$$

$$A = dh + \pi\left(\frac{d}{2}\right)^2$$

$$A = x\left(\frac{100x}{1+x}\right) + \pi\left(\frac{x}{2}\right)^2$$

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Question 4

Overview

The context of this problem is a tree, the height of which at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are provided in a table. In part (a) students were asked to use the tabular data to estimate $H'(6)$ and then to interpret the meaning of $H'(6)$, using correct units, in the context of the problem. The correct response should estimate the derivative value using a difference quotient, drawing from data in the table that most tightly bounds $t = 6$. In part (b) students were asked to explain why there must be at least one time t, for $2 < t < 10$, such that $H'(t) = 2$. A correct response should demonstrate that the Mean Value Theorem applies to H on the interval $[3, 5]$, over which the average rate of change of H (using data from the table) is $\frac{6-2}{2} = 2$. In part (c) students were asked to use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \le t \le 10$. A correct response should demonstrate that the average height of the tree for $2 \le t \le 10$ is given by dividing the definite integral of H across the interval by the width of the interval. The value of the integral $\int_2^{10} H(t)\,dt$ is to be approximated using a trapezoidal sum and data in the table. In part (d) students were given another model for the tree's height, in meters, $G(x) = \frac{100x}{1+x}$, where x is the diameter of the base of the tree, in meters. It is further given that when the tree is 50 meters tall, it is growing so that the diameter at the base of the tree is increasing at the rate of 0.03 meter per year. Using this model, students were asked to find the rate of change of the height of the tree with respect to time, in meters per year, at the time when the tree is 50 meters tall. A correct response should apply the chain rule to obtain that $\frac{dG}{dt} = \frac{dG}{dx} \cdot \frac{dx}{dt}$. The derivative expression $\frac{dG}{dx}$ can be obtained from the given expression for $G(x)$ using derivative rules (e.g., the quotient rule) and the value of $\frac{dx}{dt}$ at the instant in question provided in the problem statement.

For part (a) see LO 2.1B/EK 2.1B1, LO 2.3A/EK 2.3A1, LO 2.3A/EK 2.3A2. For part (b) see LO 2.4A/EK 2.4A1. For part (c) see LO 3.2B/EK 3.2B2, LO 3.4B/EK 3.4B1. For part (d) see LO 2.1C/EK 2.1C3, LO 2.1C/EK 2.1C4, LO 2.3C/EK 2.3C2. This problem incorporates the following Mathematical Practices for AP Calculus (MPACs): reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, connecting multiple representations, building notational fluency, and communicating.

Sample: 4A
Score: 9

The response earned all 9 points: 2 points in part (a), 2 points in part (b), 2 points in part (c), and 3 points in part (d). In part (a) the response would have earned the first point for $\frac{(11-6)}{(7-5)}$ with no simplification. In this case, correct simplification to $\frac{5}{2}$ earned the point. Although not required for the first point, the answer includes the correct units of "meters/years" which is considered for the second point. The response earned the second point for the interpretation that includes the three necessary elements: an interpretation of H' as a rate in the context of the tree, correct units of meters per year, and an interpretation of the input "6" as the moment in time of $t = 6$ years. In part (b) the response earned the first point for the difference quotient $\frac{H(5)-H(3)}{(5-3)}$ that appears in line 4. The first point does not require the substitution of function values and simplification that follows; this work is

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Question 4 (continued)

considered in the context of the second point. The response earned the second point for the explanation using the Mean Value Theorem (MVT) and the difference quotient on the interval $[3, 5]$. The response explicitly states in line 1 that $H(t)$ is continuous which is necessary to earn the point. The units displayed are correct but not required to earn either the first or second points. In part (c) the response earned the first point for the trapezoidal sum labeled "Total height" with no simplification. The response earned the second point with the boxed answer with no simplification. Note that both points were earned in part (c) without simplification of numerical answers. The units given with the average height are correct but not required to earn the second point. In part (d) the response earned the first 2 points with the correct derivative in line 2: $\dfrac{100\frac{dx}{dt}(1+x)-\left(\frac{dx}{dt}\right)100x}{(1+x)^2}$. The use of $G'(x)$ rather than $\dfrac{dG}{dt}$ notation does not impact the points earned. The response would have earned the third point for $\dfrac{100 \times 0.03(2) - 0.03 \times 100}{4}$ with no simplification. In this case, correct simplification to $\dfrac{3}{4}$ earned the third point. The response includes correct units that are not required to earn the third point.

Sample: 4B
Score: 6

The response earned 6 points: 2 points in part (a), no points in part (b), 1 point in part (c), and 3 points in part (d). In part (a) the response would have earned the first point for $\dfrac{11-6}{2}$ with no simplification. In this case, correct simplification to $\dfrac{5}{2}$ earned the point. Although not required for the first point, the answer includes the correct units of meters/year which is considered for the second point. The response earned the second point for the interpretation that includes the three necessary elements: an interpretation of H' as a rate in the context of the tree, correct units of meters per year, and an interpretation of the input "6" as the moment in time of $t = 6$ years. In part (b) the response did not earn the first point because the difference quotient does not use the interval $[3, 5]$ that results in a secant slope of 2. The response did not earn the second point because, although the Mean Value Theorem (MVT) is cited along with the continuity of H, there is no explanation connecting the Mean Value Theorem to the values of $H(t)$ in the table. In part (c) the response earned the first point for the trapezoidal sum in line 1 with no simplification. The arithmetic and simplification that follow are considered for the second point. The second point was not earned because the sum is not multiplied by $\dfrac{1}{8}$ to find the average height of the tree on the interval $[2, 10]$. In part (d) the response earned the first 2 points with the correct derivative in line 1 on the left: $\dfrac{dG}{dt} = \dfrac{(1+x)\left(100\frac{dx}{dt}\right)-(100x)\left(\frac{dx}{dt}\right)}{(1+x)^2}$. The response would have earned the third point for $\dfrac{(1+1)(100(0.03))-(100(1))(0.03)}{(1+1)^2}$ in line 2 on the left with no simplification. In this case, correct simplification to $\dfrac{3}{4}$ earned the third point. The response includes correct units that are not required to earn the third point.

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Question 4 (continued)

Sample: 4C
Score: 3

The response earned 3 points: 1 point in part (a), no points in part (b), 1 point in part (c), and 1 point in part (d). In part (a) the response would have earned the first point for $\frac{11-6}{7-5}$ with no simplification. In this case, correct simplification to $\frac{5}{2}$ earned the point. The response did not earn the second point because the interpretation of $H'(6)$ does not include an interpretation of the input "6" as the moment in time of $t = 6$ years. In part (b) the response did not earn the first point because no difference quotient is given. The response is not eligible for the second point because the explanation given does not reference the interval $[3, 5]$ that results in the secant slope of 2. Although the hypotheses of the Mean Value Theorem are stated, the conclusion is not. In part (c) the response earned the first point for the trapezoidal sum in line 1 with no simplification. The arithmetic and simplification that follow are considered for the second point. The response did not earn the second point. There is an arithmetic error in line 3, and the sum is not multiplied by $\frac{1}{8}$ to find the average height of the tree on the interval $[2, 10]$. In part (d) the response earned 1 of the first 2 points for the correct derivative of G with respect to x in line 2 on the right. Because the derivative does not include the chain rule, the response is not eligible for additional points in part (d).

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Question 5

(a) Area $= \frac{1}{2}\int_{\pi/3}^{5\pi/3}\left(4^2 - (3 + 2\cos\theta)^2\right)d\theta$

$3 : \begin{cases} 1 : \text{constant and limits} \\ 2 : \text{integrand} \end{cases}$

(b) $\frac{dr}{d\theta} = -2\sin\theta \Rightarrow \left.\frac{dr}{d\theta}\right|_{\theta=\pi/2} = -2$

$r\left(\frac{\pi}{2}\right) = 3 + 2\cos\left(\frac{\pi}{2}\right) = 3$

$y = r\sin\theta \Rightarrow \frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta$

$x = r\cos\theta \Rightarrow \frac{dx}{d\theta} = \frac{dr}{d\theta}\cos\theta - r\sin\theta$

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/2} = \left.\frac{dy/d\theta}{dx/d\theta}\right|_{\theta=\pi/2} = \frac{-2\sin\left(\frac{\pi}{2}\right) + 3\cos\left(\frac{\pi}{2}\right)}{-2\cos\left(\frac{\pi}{2}\right) - 3\sin\left(\frac{\pi}{2}\right)} = \frac{2}{3}$$

The slope of the line tangent to the graph of $r = 3 + 2\cos\theta$ at $\theta = \frac{\pi}{2}$ is $\frac{2}{3}$.

— OR —

$y = r\sin\theta = (3 + 2\cos\theta)\sin\theta \Rightarrow \frac{dy}{d\theta} = 3\cos\theta + 2\cos^2\theta - 2\sin^2\theta$

$x = r\cos\theta = (3 + 2\cos\theta)\cos\theta \Rightarrow \frac{dx}{d\theta} = -3\sin\theta - 4\sin\theta\cos\theta$

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/2} = \left.\frac{dy/d\theta}{dx/d\theta}\right|_{\theta=\pi/2} = \frac{3\cos\left(\frac{\pi}{2}\right) + 2\cos^2\left(\frac{\pi}{2}\right) - 2\sin^2\left(\frac{\pi}{2}\right)}{-3\sin\left(\frac{\pi}{2}\right) - 4\sin\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right)} = \frac{2}{3}$$

The slope of the line tangent to the graph of $r = 3 + 2\cos\theta$ at $\theta = \frac{\pi}{2}$ is $\frac{2}{3}$.

$$3 : \begin{cases} 1 : \frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta \\ \quad\text{or } \frac{dx}{d\theta} = \frac{dr}{d\theta}\cos\theta - r\sin\theta \\ 1 : \frac{dy}{dx} = \dfrac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta} \\ 1 : \text{answer} \end{cases}$$

(c) $\frac{dr}{dt} = \frac{dr}{d\theta}\cdot\frac{d\theta}{dt} = -2\sin\theta\cdot\frac{d\theta}{dt} \Rightarrow \frac{d\theta}{dt} = \frac{dr}{dt}\cdot\frac{1}{-2\sin\theta}$

$$\left.\frac{d\theta}{dt}\right|_{\theta=\pi/3} = 3\cdot\frac{1}{-2\sin\left(\frac{\pi}{3}\right)} = \frac{3}{-\sqrt{3}} = -\sqrt{3} \text{ radians per second}$$

$$3 : \begin{cases} 1 : \frac{dr}{dt} = \frac{dr}{d\theta}\cdot\frac{d\theta}{dt} \\ 1 : \frac{d\theta}{dt} = \frac{dr}{dt}\cdot\frac{1}{-2\sin\theta} \\ 1 : \text{answer with units} \end{cases}$$

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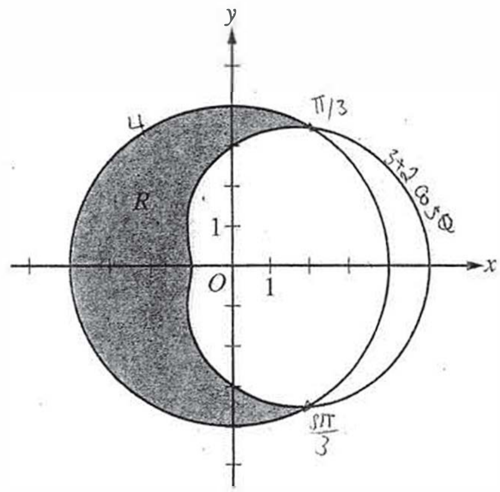

5. The graphs of the polar curves $r = 4$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect at $\theta = \dfrac{\pi}{3}$ and $\theta = \dfrac{5\pi}{3}$.

(a) Let R be the shaded region that is inside the graph of $r = 4$ and also outside the graph of $r = 3 + 2\cos\theta$, as shown in the figure above. Write an expression involving an integral for the area of R.

$$R = \frac{1}{2}\int_{\pi/3}^{5\pi/3}\left[(4)^2 - (3+2\cos(\theta))^2\right] d\theta$$

$$R = \frac{1}{2}\int_{\pi/3}^{5\pi/3}\left[16 - (3+2\cos\theta)^2\right] d\theta$$

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5A
2.f2

(b) Find the slope of the line tangent to the graph of $r = 3 + 2\cos\theta$ at $\theta = \frac{\pi}{2}$.

$r = 3+2\cos\theta$

$x = (3+2\cos\theta)\cos\theta$

$y = (3+2\cos\theta)\sin\theta$

$\frac{dy}{d\theta} = (-2\sin\theta\sin\theta) + \cos\theta(3+2\cos\theta)$

$\frac{dy}{dx} = \frac{-2\sin^2\theta + 3\cos\theta + 2\cos^2\theta}{-2\sin\theta\cos\theta - 3\sin\theta - 2\sin\theta\cos\theta}$

$\frac{dx}{d\theta} = (-2\sin\theta\cos\theta) - \sin\theta(3+2\cos\theta)$

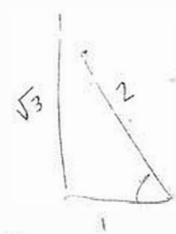
At $\frac{\pi}{2}$

$\frac{-2(1)+0+0}{-2(1)(0)-3(1)-0} = \frac{-2}{-3}$

$\frac{dy}{dx}$ $\boxed{=\frac{2}{3}}$

(c) A particle moves along the portion of the curve $r = 3 + 2\cos\theta$ for $0 < \theta < \frac{\pi}{2}$. The particle moves in such a way that the distance between the particle and the origin increases at a constant rate of 3 units per second. Find the rate at which the angle θ changes with respect to time at the instant when the position of the particle corresponds to $\theta = \frac{\pi}{3}$. Indicate units of measure.

$\frac{dr}{dt} = -2\sin\theta\frac{d\theta}{dt}$



$\frac{dr}{dt} = 3$

$3 = -2\sin\theta\frac{d\theta}{dt}$

$\frac{\sqrt{3}}{2}$

$-\frac{3}{2} = \sin\theta\frac{d\theta}{dt}$

At $\frac{\pi}{3}$ $\quad \frac{-\frac{3}{2}}{\sin\frac{\pi}{3}} = \frac{d\theta}{dt} \quad \frac{-\frac{3}{2}}{\frac{\sqrt{3}}{2}} = \frac{d\theta}{dt} \quad \frac{-6}{2\sqrt{3}} = \boxed{-\sqrt{3} \text{ radians per second}}$

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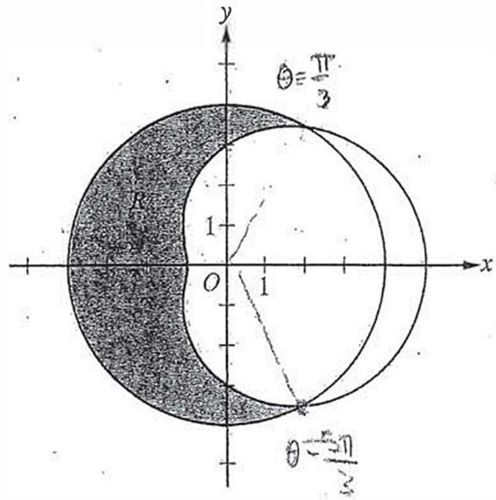

5. The graphs of the polar curves $r = 4$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect at $\theta = \frac{\pi}{3}$ and $\theta = \frac{5\pi}{3}$.

(a) Let R be the shaded region that is inside the graph of $r = 4$ and also outside the graph of $r = 3 + 2\cos\theta$, as shown in the figure above. Write an expression involving an integral for the area of R.

$$\frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} [4 - 3 + 2\cos\theta]\,d\theta$$

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(b) Find the slope of the line tangent to the graph of $r = 3 + 2\cos\theta$ at $\theta = \frac{\pi}{2}$.

$y = r\sin\theta = 3\sin\theta + (2\cos\theta)\sin\theta$

$x = r\cos\theta = 3\cos\theta + 2\cos^2\theta$

$$\frac{dy}{dx} = \frac{3\cos\theta + 2\cos^2\theta - 2\sin^2\theta}{-3\sin\theta - 4\cos\theta\sin\theta}\Bigg|_{\theta=\frac{\pi}{2}} = \frac{0+0-2}{-3-0}$$

$$\frac{dy}{dx} = \frac{2}{3}$$

(c) A particle moves along the portion of the curve $r = 3 + 2\cos\theta$ for $0 < \theta < \frac{\pi}{2}$. The particle moves in such a way that the distance between the particle and the origin increases at a constant rate of 3 units per second. Find the rate at which the angle θ changes with respect to time at the instant when the position of the particle corresponds to $\theta = \frac{\pi}{3}$. Indicate units of measure.

$\frac{dr}{dt} = 3 \quad \frac{d\theta}{dt} = ? \quad \theta = \frac{\pi}{3} \quad r = 4$

$$\frac{dr}{dt} = -2\sin\theta\,\frac{d\theta}{dt}$$

$$3 = -2\left(\frac{1}{2}\right)\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = -3 \text{ rad/sec}$$

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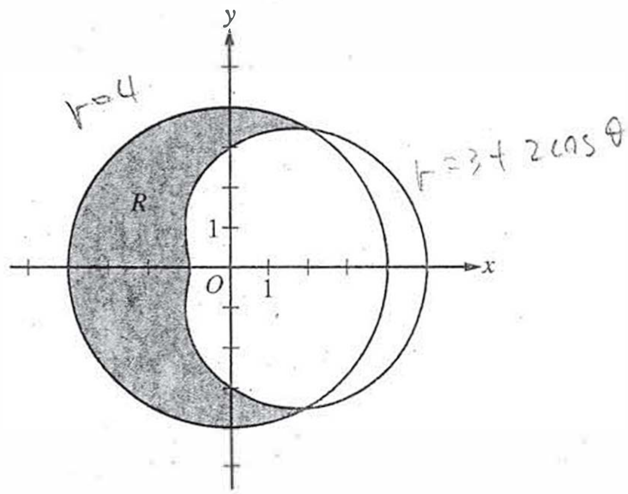

5. The graphs of the polar curves $r = 4$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect at $\theta = \frac{\pi}{3}$ and $\theta = \frac{5\pi}{3}$.

(a) Let R be the shaded region that is inside the graph of $r = 4$ and also outside the graph of $r = 3 + 2\cos\theta$, as shown in the figure above. Write an expression involving an integral for the area of R.

$$\frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} \left(4 - (3+2\cos\theta)\right)^2 \, d\theta$$

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(b) Find the slope of the line tangent to the graph of $r = 3 + 2\cos\theta$ at $\theta = \frac{\pi}{2}$.

$\frac{dr}{d\theta} = -2\sin\theta \Big|_{\theta=\frac{\pi}{2}}$

$= \underline{\underline{-2}}$

(c) A particle moves along the portion of the curve $r = 3 + 2\cos\theta$ for $0 < \theta < \frac{\pi}{2}$. The particle moves in such a way that the distance between the particle and the origin increases at a constant rate of 3 units per second. Find the rate at which the angle θ changes with respect to time at the instant when the position of the particle corresponds to $\theta = \frac{\pi}{3}$. Indicate units of measure.

$\frac{dr}{dt} = 3$

$\frac{dr}{dt} = -2\sin\theta \frac{d\theta}{dt}$

$3 = -2\sin\frac{\pi}{3} \frac{d\theta}{dt}$

$3 = -\sqrt{3} \frac{d\theta}{dt}$

$\frac{d\theta}{dt} = -\frac{3}{\sqrt{3}}$

$= -\frac{9}{3} = \boxed{-3 \text{ rad/sec}}$

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Question 5

Overview

In this problem a polar graph is provided for polar curves $r = 4$ and $r = 3 + 2\cos\theta$. It was given that the curves intersect at $\theta = \frac{\pi}{3}$ and $\theta = \frac{5\pi}{3}$. In part (a) students were asked for an integral expression that gives the area of the region R that is inside the graph of $r = 4$ and outside the graph of $r = 3 + 2\cos\theta$. A correct response should resource the formula for the area of a simple polar region as half of a definite integral of the square of the radius function. The area of R is given by $\frac{1}{2}\int_{\pi/3}^{5\pi/3} 4^2\, d\theta - \frac{1}{2}\int_{\pi/3}^{5\pi/3}(3 + 2\cos\theta)^2\, d\theta = \frac{1}{2}\int_{\pi/3}^{5\pi/3}\left(4^2 - (3 + 2\cos\theta)^2\right) d\theta.$

In part (b) students were asked for the slope of the line tangent to the graph of $r = 3 + 2\cos\theta$ at $\theta = \frac{\pi}{2}$. A correct response should deal with the conversion between polar and rectangular coordinate systems given by $y = r\sin\theta$ and $x = r\cos\theta$, differentiate these with respect to θ using the product rule, and find the slope of the line tangent to the graph as the value of $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ at $\theta = \frac{\pi}{2}$. In part (c) the motion of a particle along the portion of the curve $r = 3 + 2\cos\theta$ for $0 < \theta < \frac{\pi}{2}$ is such that the distance between the particle and the origin increases at a constant rate of 3 units per second. Students were asked for the rate at which the angle θ changes with respect to time at the instant when the position of the particle corresponds to $\theta = \frac{\pi}{3}$ and to indicate units of measure. A correct response should use the chain rule to relate the rates of r and θ with respect to time t: $\frac{dr}{dt} = -2\sin\theta \cdot \frac{d\theta}{dt}$. Recognizing that $\frac{dr}{dt} = 3$ from the problem statement, it follows that $\left.\frac{d\theta}{dt}\right|_{\theta=\pi/3} = -\sqrt{3}$ radians per second.

For part (a) see LO 3.4D/EK 3.4D1 (BC). For part (b) see LO 2.1C/EK 2.1C7 (BC), LO 2.2A/EK 2.2A4 (BC), LO 2.3B/EK 2.3B1. For part (c) see LO 2.1C/EK 2.1C7 (BC), LO 2.2A/EK 2.2A4 (BC), LO 2.3C/EK 2.3C2. This problem incorporates the following Mathematical Practices for AP Calculus (MPACs): reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, connecting multiple representations, building notational fluency, and communicating.

Sample: 5A
Score: 9

The response earned all 9 points: 3 points in part (a), 3 points in part (b), and 3 points in part (c). In part (a) the response earned the first point with the constant of $\frac{1}{2}$ and the limits of integration of $\frac{\pi}{3}$ and $\frac{5\pi}{3}$ in line 1. The response would have earned the 2 integrand points with $\left[(4)^2 - (3 + 2\cos(\theta))^2\right]$ in line 1 with no simplification. In this case, the correct simplification in line 2 earned the points. In part (b) the response earned the first point for one of the following: $\frac{dx}{d\theta}$ is computed in line 5 on the left, and $\frac{dy}{d\theta}$ is computed in line 4 on the left. The response earned the second point with the assembly of $\frac{dy}{dx}$ in line 1 on the right. The response would have earned the third point with the expression $\frac{-2(1) + 0 + 0}{-2(1)(0) - 3(1) - 0}$ on the right with no simplification. In this

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case, the correct simplification to $\frac{2}{3}$ earned the point. In part (c) the response earned the first point for application of the chain rule with $\frac{dr}{dt} = -2\sin\theta\frac{d\theta}{dt}$ in line 1. Although the response substitutes 3 for $\frac{dr}{dt}$ and $\frac{\pi}{3}$ for θ before isolating $\frac{d\theta}{dt}$, the equation $\frac{-\frac{3}{2}}{\sin\frac{\pi}{3}} = \frac{d\theta}{dt}$ in the last line on the left earned the second point. The response earned the third point with $-\sqrt{3}$ radians per second in the last line on the right.

Sample: 5B
Score: 6

The response earned 6 points: 1 point in part (a), 3 points in part (b), and 2 points in part (c). In part (a) the response earned the first point with the constant of $\frac{1}{2}$ and the limits of integration of $\frac{\pi}{3}$ and $\frac{5\pi}{3}$ in line 1. The response did not earn either of the integrand points because the integrand is incorrect. In part (b) the response earned the first point for one of the following: $\frac{dy}{d\theta}$ is computed in the numerator of line 3, and $\frac{dx}{d\theta}$ is computed in the denominator of line 3. The response earned the second point with the assembly of $\frac{dy}{dx}$ in line 3. The response would have earned the third point with the expression $\frac{0+0-2}{-3-0}$ in line 3 with no simplification. In this case, the correct simplification to $\frac{2}{3}$ earned the point. In part (c) the response earned the first point for application of the chain rule with $\frac{dr}{dt} = -2\sin\theta\frac{d\theta}{dt}$ in line 2. The response earned the second point because although $\sin\frac{\pi}{3}$ is incorrectly evaluated as $\frac{1}{2}$ in line 3, $\frac{d\theta}{dt}$ is isolated correctly with a consistent result in the equation $\frac{d\theta}{dt} = -3$ in the last line. The response did not earn the third point because the evaluation of $\sin\frac{\pi}{3}$ as $\frac{1}{2}$ makes the response not eligible for the third point. The units are correct.

Sample: 5C
Score: 3

The response earned 3 points: 1 point in part (a), no points in part (b), and 2 points in part (c). In part (a) the response earned the first point with the constant of $\frac{1}{2}$ and the limits of integration of $\frac{\pi}{3}$ and $\frac{5\pi}{3}$. The response did not earn either of the integrand points because the integrand is incorrect. In part (b) the response did not earn any points. The response did not earn the first point because neither $\frac{dx}{d\theta}$ nor $\frac{dy}{d\theta}$ is presented. Therefore, the response is not eligible for the second and third points. In part (c) the response earned the first point for application of the chain rule with $\frac{dr}{dt} = -2\sin\theta\frac{d\theta}{dt}$ in line 1 on the right. Although the response substitutes 3 for

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Question 5 (continued)

$\frac{dr}{dt}$ and $\frac{\pi}{3}$ for θ before isolating $\frac{d\theta}{dt}$, the equation $\frac{d\theta}{dt} = -\frac{3}{\sqrt{3}}$ in line 4 on the right earned the second point. The response did not earn the third point because the answer is incorrect. The units are correct.

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Question 6

(a) The first four nonzero terms are $\frac{x^2}{3} - \frac{x^3}{2 \cdot 3^2} + \frac{x^4}{3 \cdot 3^3} - \frac{x^5}{4 \cdot 3^4}$.

The general term is $(-1)^{n+1} \frac{x^{n+1}}{n \cdot 3^n}$.

$2 : \begin{cases} 1 : \text{first four terms} \\ 1 : \text{general term} \end{cases}$

(b) $$\lim_{n \to \infty} \left| \frac{\frac{(-1)^{n+2} x^{n+2}}{(n+1)\left(3^{n+1}\right)}}{\frac{(-1)^{n+1} x^{n+1}}{n \cdot 3^n}} \right| = \lim_{n \to \infty} \left| \frac{-x}{3} \cdot \frac{n}{(n+1)} \right| = \left| \frac{x}{3} \right|$$

$\left| \frac{x}{3} \right| < 1$ for $|x| < 3$

Therefore, the radius of convergence of the Maclaurin series for f is 3.

$5 : \begin{cases} 1 : \text{sets up ratio} \\ 1 : \text{computes limit of ratio} \\ 1 : \text{radius of convergence} \\ 1 : \text{considers both endpoints} \\ 1 : \text{analysis and interval of convergence} \end{cases}$

— OR —

The radius of convergence of the Maclaurin series for $\ln(1 + x)$ is 1, so the series for $f(x) = x \ln\left(1 + \frac{x}{3}\right)$ converges absolutely for $\left| \frac{x}{3} \right| < 1$.

$\left| \frac{x}{3} \right| < 1 \Rightarrow |x| < 3$

Therefore, the radius of convergence of the Maclaurin series for f is 3.

— OR —

$5 : \begin{cases} 1 : \text{radius for } \ln(1+x) \text{ series} \\ 1 : \text{substitutes } \frac{x}{3} \\ 1 : \text{radius of convergence} \\ 1 : \text{considers both endpoints} \\ 1 : \text{analysis and interval of convergence} \end{cases}$

When $x = -3$, the series is $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(-3)^{n+1}}{n \cdot 3^n} = \sum_{n=1}^{\infty} \frac{3}{n}$, which diverges by comparison to the harmonic series.

When $x = 3$, the series is $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3^{n+1}}{n \cdot 3^n} = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{3}{n}$, which converges by the alternating series test.

The interval of convergence of the Maclaurin series for f is $-3 < x \le 3$.

(c) By the alternating series error bound, an upper bound for $|P_4(2) - f(2)|$ is the magnitude of the next term of the alternating series.

$|P_4(2) - f(2)| < \left| -\frac{2^5}{4 \cdot 3^4} \right| = \frac{8}{81}$

$2 : \begin{cases} 1 : \text{uses fifth-degree term as error bound} \\ 1 : \text{answer} \end{cases}$

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6. The Maclaurin series for $\ln(1+x)$ is given by

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n+1}\frac{x^n}{n} + \cdots.$$

On its interval of convergence, this series converges to $\ln(1+x)$. Let f be the function defined by $f(x) = x\ln\left(1+\frac{x}{3}\right)$.

(a) Write the first four nonzero terms and the general term of the Maclaurin series for f.

$$f(x) \approx \frac{x^2}{3} - \frac{x\left(\frac{x}{3}\right)^2}{2} + \frac{x\left(\frac{x}{3}\right)^3}{3} - \frac{x\left(\frac{x}{3}\right)^4}{4}$$

$$\approx \frac{x^2}{3} - \frac{\frac{x^3}{9}}{2} + \frac{\frac{x^4}{27}}{3} - \frac{\frac{x^5}{81}}{4} + \ldots + (-1)^{n+1}\frac{\frac{x^{n+1}}{3^n}}{n}$$

$$\approx \frac{x^2}{3} - \frac{x^3}{9\cdot 2} + \frac{x^4}{27\cdot 3} - \frac{x^5}{81\cdot 4} + \ldots + (-1)^{n+1}\frac{x^{n+1}}{n3^n}$$

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6A 2+2

(b) Determine the interval of convergence of the Maclaurin series for f. Show the work that leads to your answer.

Ratio Test: $\lim_{n\to\infty}\left|\frac{(-1)^{n+2}x^{n+2}}{(n+1)3^{n+1}}\cdot\frac{n3^n}{(-1)^{n+1}x^{n+1}}\right|$

$\lim_{n\to\infty}\left|\frac{-nx}{3(n+1)}\right|$

$\left|\frac{x}{3}\right|\lim_{n\to\infty}\left|\frac{n}{n+1}\right|$

$\left|\frac{x}{3}\right|\cdot 1 < 1$

$\left|\frac{x}{3}\right| < 1$

$-1 < \frac{x}{3} < 1$

$-3 < x < 3$

3: $\frac{(-1)^{n+1}\cdot 3^{n+1}}{n3^n} = \frac{3(-1)^{n+1}}{n}$

alternating harmonic ∴ converges conditionally

-3: $\frac{(-1)^{n+1}(-3)^{n+1}}{n3^n} = \frac{(-1)^{2n+2}(3)^{n+1}}{n3^n}$

$= \frac{3}{n}$ harmonic ∴ diverges

Interval: $-3 < x \leq 3$

(c) Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. Use the alternating series error bound to find an upper bound for $|P_4(2) - f(2)|$.

$|P_4(2) - f(2)| <$

$= \left|\frac{x^5}{81\cdot 4}\right|$

$= \left|\frac{2^5}{81\cdot 4}\right|$

$\frac{32}{81\cdot 4} = \frac{8}{81}$

$|P_4(2) - f(2)| < \frac{8}{81}$

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6. The Maclaurin series for $\ln(1+x)$ is given by

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n+1}\frac{x^n}{n} + \cdots.$$

On its interval of convergence, this series converges to $\ln(1+x)$. Let f be the function defined by $f(x) = x\ln\left(1+\frac{x}{3}\right)$.

(a) Write the first four nonzero terms and the general term of the Maclaurin series for f.

$$x\ln(1+x) = x^2 - \frac{x^3}{2} + \frac{x^4}{3} - \frac{x^5}{4} + \ldots + \frac{(-1)^{n+1}x^{n+1}}{n}$$

$$x\ln\left(1+\frac{x}{3}\right) = \left(\frac{x}{3}\right)^2 - \frac{1}{2}\left(\frac{x}{3}\right)^3 + \frac{1}{3}\left(\frac{x}{3}\right)^4 - \frac{1}{4}\left(\frac{x}{3}\right)^5$$

$$= \boxed{\frac{x^2}{9} - \frac{x^3}{2\cdot 27} + \frac{x^4}{3\cdot 81} - \frac{x^5}{4\cdot 243} + \ldots + \frac{(-1)^{n+1}\left(\frac{x}{3}\right)^{n+1}}{n}}$$

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(b) Determine the interval of convergence of the Maclaurin series for f. Show the work that leads to your answer.

$f(x) = x\ln\left(1+\frac{x}{3}\right)$

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(-1)^{n+2}\left(\frac{x}{3}\right)^{n+2}}{n+1}\cdot\frac{n}{\cancel{(-1)^{n+1}}\cdot\left(\frac{x}{3}\right)^{n+1}}\right| = \left|\frac{x}{3}\right|$

$\left|\frac{x}{3}\right| < 1 \qquad |x| < 3 \qquad x = 3$

$x = -3$

$\boxed{-3 < x \leq 3}$

$x = 3 \qquad \frac{(-1)^{n+1}(1)^{n+1}}{n+1}$ converges

$x = -3 \qquad \frac{(-1)^{n+1}(-1)^{n+1}}{n+1}$ diverges

(c) Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. Use the alternating series error bound to find an upper bound for $|P_4(2) - f(2)|$.

$|P_4(2) - f(2)| < \left|\frac{-(2)^5}{4\cdot 243}\right|$

$\boxed{\text{upper bound is } \frac{2^5}{4\cdot 243}}$

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6. The Maclaurin series for $\ln(1 + x)$ is given by

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n+1}\frac{x^n}{n} + \cdots.$$

On its interval of convergence, this series converges to $\ln(1 + x)$. Let f be the function defined by $f(x) = x\ln\left(1 + \frac{x}{3}\right)$.

(a) Write the first four nonzero terms and the general term of the Maclaurin series for f. H

$$f(x) = \frac{x^2}{3} - \frac{x^3}{6} + \frac{x^4}{9} - \frac{x^5}{12} + \ldots + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{n+1}}{3n}$$

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(b) Determine the interval of convergence of the Maclaurin series for f. Show the work that leads to your answer.

$$\lim_{n\to\infty}\left|\frac{\frac{(-1)^{n+2}x^{n+2}}{3n+3}}{\frac{(-1)^{n+1}(x)^{n+1}}{3n}}\right| = \lim_{n\to\infty}\left|\frac{\cancel{(-1)^{n+2}}\cancel{(x)^{n+2}}\cdot(3n)}{\cancel{(-1)^{n+1}}\cancel{(x)^{n+1}}(3n+3)}\right|$$

$$=\lim_{n\to\infty}\left|\frac{-x\,3n}{3n+3}\right| = |-x|$$

$$|-x|<1$$

$$\boxed{-1<x\le 1}$$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-1)^{n+1}}{3n}$$ harmonic diverges

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\cancel{(1)^{n+1}}}{3n}$$ converges alt. harmonic

(c) Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. Use the alternating series error bound to find an upper bound for $|P_4(2) - f(2)|$.

$$\left|-\frac{x^4}{4}-\frac{x^5}{12}\right| \le \frac{x^6}{15}$$

$$\left|-4-\frac{8}{3}\right| \le \frac{64}{15}$$

$$\left|\frac{-16}{3}\right| \le \frac{64}{15}$$

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Question 6

Overview

In this problem the first four nonzero terms and the general term of the Maclaurin series for $\ln(1 + x)$ are given, and the function f is defined by $f(x) = x\ln\left(1 + \frac{x}{3}\right)$. In part (a) students were asked for the first four nonzero terms and the general term of the Maclaurin series for f. A correct response should substitute $\frac{x}{3}$ for x in the supplied terms of the series for $\ln(1 + x)$, multiply the resulting terms by x, and expand so that each term is a constant multiple of a power of x. The general term should also be included. In part (b) students were asked to determine the interval of convergence of the Maclaurin series for f with supporting work for their answer. A correct response should demonstrate the use of the ratio test to determine the radius of convergence of the series and, then, a test of the endpoints of the interval of convergence to determine which endpoints, if any, are to be included in the interval of convergence. In part (c) students were asked to use the alternating series error bound to find an upper bound for $|P_4(2) - f(2)|$, where $P_4(x)$ is the fourth-degree Taylor polynomial for f about $x = 0$. A correct response should indicate that the alternating series error bound bounds $|P_4(2) - f(2)|$ by the magnitude of the next term in the alternating series formed by evaluating the Taylor series for f about $x = 0$ at $x = 2$.

For part (a) see LO 4.2B/EK 4.2B5. For part (b) see LO 4.1A/EK 4.1A3, LO 4.1A/EK 4.1A6, LO 4.2C/EK 4.2C2. For part (c) see LO 4.2A/EK 4.2A5. This problem incorporates the following Mathematical Practices for AP Calculus (MPACs): reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, building notational fluency, and communicating.

Sample: 6A
Score: 9

The response earned all 9 points: 2 points in part (a), 5 points in part (b), and 2 points in part (c). In part (a) the response would have earned the first point in line 2 for the first four nonzero terms of the Maclaurin series for f with no simplification. In this case, the simplification is correct, and the point was earned in line 3. The response would have earned the second point in line 2 with the general term of the Maclaurin series for f with no simplification. In this case, the simplification is correct, and the point was earned in line 3. In part (b) the response earned the first point with the ratio in line 1 on the left. The response earned the second point in line 4 on the left with $\left|\frac{x}{3}\right|$. The response earned the third point in line 7 on the left with $-3 < x < 3$. The response earned the fourth point with the work on the right by substituting the endpoints $x = 3$ and $x = -3$ into the general term of the Maclaurin series for f. The response earned the fifth point for the work on the right with the analysis — for $x = 3$ stating "alternating harmonic $\therefore$ converges conditionally" and for $x = -3$ stating "harmonic $\therefore$ diverges" — and the interval of convergence in the last line on the right. In part (c) the response earned the first point in line 1 on the right with use of the fifth-degree term. The response earned the second point by substituting 2 for x in the fifth-degree term and taking an absolute value to guarantee that a positive error bound is returned.

Sample: 6B
Score: 6

The response earned 6 points: no points in part (a), 4 points in part (b), and 2 points in part (c). In part (a) the response did not earn the first point as the denominators are incorrect in all of the first four nonzero terms of the Maclaurin series for f. The response did not earn the second point because of an incorrect general term of the Maclaurin series for f. In part (b) the response earned the first point for a ratio in line 2 that is consistent with the

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Question 6 (continued)

general term presented in part (a). The response earned the second point in line 2 with $\left|\frac{x}{3}\right|$. The response earned the third point with $|x| < 3$ in line 3 and a radius of convergence of the Maclaurin series for f that is consistent with the general term presented in part (a). The response earned the fourth point by substituting the endpoints $x = 3$ and $x = -3$ into the imported general term from part (a). The response did not earn the fifth point as the response states "converges" and "diverges" without justification. In part (c) the response earned both points with an expression for the absolute value of the fifth-degree term evaluated at $x = 2$ that is consistent with the fifth-degree term of the series in part (a).

Sample: 6C
Score: 3

The response earned 3 points: no points in part (a), 3 points in part (b), and no points in part (c). In part (a) the response did not earn the first point as the response has incorrect denominators in the second, third, and fourth terms of the Maclaurin series for f. The response did not earn the second point because of an incorrect general term of the Maclaurin series for f. In part (b) the response earned the first point for a ratio given in the top left corner that is consistent with the general term presented in part (a). The incorrect general term imported from part (a) results in an oversimplification of the question, so the response is not eligible for the second and third points. The response earned the fourth point by substituting $x = -1$ and $x = 1$ into the general term imported from part (a). The response earned the fifth point with the analysis — for $x = -1$ with "harmonic diverges" and for $x = 1$ with "converges alt. harmonic"— and the interval of convergence boxed on the left that is consistent with the general term presented in part (a). In part (c) the response did not earn the first point because the sixth-degree term is used to find the requested upper bound instead of the fifth-degree term. The response did not earn the second point because the error bound is not consistent with the presented fifth-degree term in part (a).

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